ELSEVIER

Comparison of aiding and opposing mixed convection heat transfer in a vertical tube with Grashof number variation

Donald D. Joye
Department of Chemical Engineering, Villanova University, Villanova, PA, USA

Mixed convection heat transfer in a vertical tube with aiding and opposing flows (upflow and downflow heating) was studied experimentally for Reynolds numbers ranging from 700 to 25,000 at constant Grashof number under constant wall temperature (CWT) conditions. The Grashof number was varied independently by adjusting steam pressure in the jacket of a vertical, double-pipe heat exchanger. The mixed convection region existed at Re between about 4,000 and 10,000 (this range was Gr dependent). As the Grashof number was reduced in this Re range, the Nusselt number was also reduced. The data show that the way in which Nu is reduced is not identical for aiding and opposing flows. Comparison of the present data with literature on the basis of buoyancy parameter show good agreement, except for the asymptote region.

Keywords: vertical mixed convection; heat transfer enhancement; Grashof number variation

Introduction

When natural and forced convection heat transfer mechanisms interact, "combined" or "mixed" convection is said to exist. The buoyancy force acts vertically, but the forced flow may be in any direction. The interactions of natural and forced convection currents are very complex, and each case of geometry and orientation must be treated separately. In vertical, internal flows, the forced flow may be upward or downward, and the heat transfer may be to or from the fluid in the conduit. Heating in upflow is termed "aiding" flow, because the natural convection currents are in the same direction as the forced flow. Downflow heating is termed "opposing" flow, because the buoyancy currents are opposite. This work deals with heating of fluid in a vertical tube with forced flow in both upward and downward directions. In a companion paper, (Joye 1995) the downflow heating results are compared with the correlations of Jackson et al. (1989) and Swanson and Catton (1987). No complete correlation presently exists for the upflow heating situation, although several correlations exist for limited ranges of data.

Vertical internal flow heat transfer has application to chemical process heat transfer, nuclear power technology (Symolu et al. 1987; Ianello et al. 1988) and some aspects of electronic cooling. More recently, some have recognized that significant heat transfer enhancement may be realized in vertical flow situations, and this is already driving research on vertical, unbaffled shell-and-tube heat exchangers. An extensive review paper by Jackson, et al. (1989) summarizes most of the work in this field.

In an experimental study of heat transfer characteristics for flow in a vertical tube, Joye et al. (1989) present data for upflow and downflow heating at high, but single-valued, Grashof number with constant wall temperature (CWT) boundary condition. The other rigorous boundary condition (uniform heat flux at the wall, or UHF) is more amenable to theory and experimental control than the CWT condition; hence, it has been more popular for study. Both boundary conditions have practical importance, but CWT is more common in process industries, mostly because of condensing vapors or boiling liquids.

Three regions characterize heat transfer in CWT situations. The turbulent region occurs at Reynolds numbers above about 10,000 (depending on Grashof number), and the results for vertical mixed convection heat transfer, whether upflow or downflow, heating or cooling, CWT or UHF boundary condition, agree well with forced flow correlations; e.g., the Sieder–Tate or any of the newer relationships (Holman 1990). Mixed convection is not important in the turbulent region above this upper transition point, because turbulence at this level is much stronger than the natural convection mechanism and dominates the heat transfer. At the other extreme, the asymptote region occurs at low Reynolds numbers (about 4,000 and below). This region results from the flow being slow enough so that the outlet temperature approaches the wall temperature. This situation has been described previously by Martinelli et al. (1942) and McAdams (1954) for the CWT boundary condition.

At Reynolds numbers between the upper and lower transition points, the "laminar" mixed convection region exists, where both natural convection and forced convection mechanisms are the same order of magnitude and interact in complex ways. The term "laminar" for this region is in quotes to distinguish it from

Address reprint requests to Prof. D. D. Joye, Department of Chemical Engineering, Villanova University, Villanova, PA 19085-1681, USA.

Received 10 March 1995; accepted 23 August 1995

Int. J. Heat and Fluid Flow 17: 96–101, 1996

655 Avenue of the Americas, New York, NY 10010

0142-727X/96/$15.00
SSDI 0142-727X(95)00077-1

the fully turbulent region where buoyancy is not important. Flow in this region is not, hydrodynamically speaking, laminar, because the Reynolds numbers are above 2100. At the entrance to the tube, the flow is turbulent. When aiding flow exists, many investigators prefer to call this region a "re-laminarized" flow or a "stabilized laminar flow" owing to velocity profile distortion and suppression of turbulence. From many investigations, the nature of the flow appears to be laminar, indeed. When the flow is opposing, however, there is evidence of turbulence throughout the flow field because of shear at the boundary layer, reverse flow or backflow in the boundary layer near the wall and other reasons depending on Gr and Re. The flow here is not laminar; however, the heat transfer characteristics also lie between the same two transition points. Perhaps "mixed convection" should be sufficient to describe this region for both aiding and opposing flows, but old terminology is hard to let go.

The transitions between the three regions are affected by several factors. Marcucci and Joye (1985) showed these transitions are L/D dependent, but the dependence is not strong when Gr based on diameter instead of length is used. The transitions must also depend on Grashof number. The results for upflow heating are significantly different than those for downflow heating, and these are commonly recognized as two separate cases.

Most of the experimental literature shows limited study or restricted range of parameters because of experimental difficulties in controlling the Grashof number. This is especially true with constant wall temperature boundary condition, but there are thermal limitations with the uniform heat flux boundary condition as well. Thus, one object of the present work is to develop further a database that includes Grashof number as a parameter with as wide a variation as possible for both upflow and downflow heating under CWT conditions.

Experimental method

The apparatus employed for the present study was identical to that of Joye et al., (1989) except that the steam condensing in the annulus of a copper–copper, double-pipe heat exchanger was routed to a Nash vacuum pump, and a control system was installed on the steam line to control steam pressure and its distribution in the jacket. Figure 1 shows the detail of the exchanger. A vacuum gauge measured the (vacuum) pressure in the jacket, and a pressure gauge measured positive steam pressure when that was used. Because five thermocouples were used to measure wall temperature, the vacuum pressure itself had little significance other than being constant for a given run. The vacuum pressures in the jacket were 99.6, 67.4, 33.6, and 30.2 kPa absolute pressure (0.5, 10, 20, and 21 in Hg vacuum, respectively). A run at 205 kPa (15 psig) steam pressure was included to extend the range of the present data. Any particular steam pressure value was easily held constant by the control system. Very probably, steam flowed through the jacket as well as condensed in it, but this does not affect the results, because the wall temperatures were measured directly. Constant wall temperature could be maintained and checked as well as in the previous investigation (Joye et al. 1989). At low pressures, the steam can often condense at the inlet only, and when this happened, the steam valves were adjusted to provide sufficient flow so that constant wall temperature was evident by readings on the digital thermometer. The lowest pressures were at the limit of the apparatus capability. Wall temperatures and, hence, Grashof numbers could be changed easily by adjusting the steam pressure within the limits of the equipment.

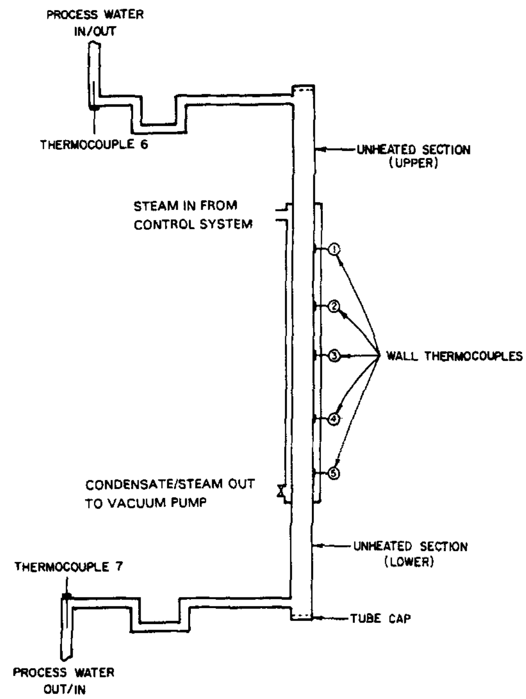

Figure 1 Experimental apparatus; test section detail

Copper–constantan theromocouples were used as before to measure both wall temperature and inlet/outlet temperatures. The unit was insulated to minimize heat loss, particularly impor-

Notation

C_p	fluid heat capacity, kJ/(kg – K)
D	tube diameter, m
g	gravitational acceleration, m/s^2
Gr_{bD}	Grashof number based on fluid properties evaluated at the average *bulk* temperature and tube diameter, $\rho^2 g D^3 \beta \Delta T/\mu^2$
Gr_{fD}	Grashof number based on fluid properties evaluated at the average *film* temperature and tube diameter, $\rho^2 g D^3 \beta \Delta T/\mu^2$
Gr_q	modified Grashof number based on heat flux, $\rho^2 g D^4 \beta q/k\mu^2$
Gz	Graetz number, wC_p/kL
h	film heat transfer coefficient, $W/(m^2 - K)$
k	fluid thermal conductivity, W/(m – K)
L	heated length of tube, m
Nu	Nusselt number, hD/k
Pr	Prandtl number, $\mu C_p/k$
Re	Reynolds number, $Dv\rho/\mu$
q	heat flux, W/m^2
T	Temperature, K
ΔT	Temperature difference (K) average wall to average bulk fluid for Grashof number, unless otherwise noted
w	mass flow rate, kg/s
v	average fluid velocity, m/s

Greek

β	volume expansivity, 1/K
μ	viscosity, Pa · s
ρ	density, kg/m^3

tant at the outlet. Mixing elbows were used to get a well-mixed outlet temperature in both upflow and downflow. Two rotameters were used to measure the flow rates at high throughput and at low throughput. The L/D of the heated section was 49.6; the inside diameter of the central tube was 0.032 m (1.265 in.). Water in the tube was prevented from boiling or degassing by pump pressures of 150–200 kPa gauge (20–30 psig) as previously (Joye et al. 1989). In both studies, the heated section was preceded by a length of tube equivalent to about 14 diameters. This is not critical, because the experiment yields average parameters, not local information.

Heat transfer data including all dimensionless groups, coefficients, etc. were calculated on spreadsheet software. Heat transfer coefficients (film coefficients) were calculated from Newton's Law of Cooling, where the heat rate (W or Btu/hr) was calculated from the temperature rise and the flow rate of water, and an arithmetic average temperature driving force was used. The Nusselt number was based on the arithmetic average (bulk fluid) temperature, and the Grashof number was based on the film temperature (arithmetic average of wall and bulk average temperature). This was chosen to represent best the actual temperature of the fluid in the film region close to the wall where the buoyancy effects exist. Others have used the wall temperature or the bulk average temperature, both of which have their advantages. Jackson et al. (1989) prefer a Grashof number defined by average density in the film, but they do not do the same for viscosity. The Grashof numbers of the present work could be converted to other bases, because all the appropriate temperatures were known.

Results and discussion

Water was used as the test fluid, and the Prandtl number (an uncontrolled parameter) varied from about 3 at low Reynolds numbers to about 9 at high Reynolds numbers. The corresponding viscosity ratio, ϕ_v = the viscosity at the bulk average temperature divided by that at the wall temperature, varied from 1.5 to 4.0. The Grashof numbers based on diameter and fluid properties at the film temperature were constant for each run to within about $\pm 20\%$. Grashof numbers at Reynolds numbers greater than about 12,000 differed from the average for that run by greater than 20%, but this does not matter, because the effect of Gr in this range (turbulent flow) is negligible. Table 1 gives the average Grashof number for each run at different temperature bases as a function of shell-side steam pressure. The upflow/downflow Grashof numbers differed by less than 5%, so the Grashof numbers for corresponding upflow and downflow conditions are essentially identical. The corresponding Grashof number based on length can be computed easily, because $L/D = 49.6$ for all cases. With reference to the table, the Grashof numbers calculated by the Jackson et al. (1989) method are about 60% of the Gr based on fluid properties evaluated at the bulk average temperature and almost an order of magnitude less than Gr based on fluid properties taken at the film temperature.

Table 1 Grashof number as function of (absolute) steam pressure in the shell

Steam P(abs)	Gr_{fD}	Gr_{bD}	Gr_{barD}
205 kPa	1.00×10^{8}	3.20×10^{7}	2.21×10^{7}
99.6 kPa	7.65×10^{7}	1.95×10^{7}	1.04×10^{7}
67.4 kPa	3.15×10^{7}	0.80×10^{7}	0.65×10^{7}
33.6 kPa	1.45×10^{7}	0.48×10^{7}	0.29×10^{7}
30.2 kPa	1.00×10^{7}	0.32×10^{7}	0.24×10^{7}

* Gr calculated on film temperture basis (Gr_{fD}), bulk average temperature basis (Gr_{bD}) and Jackson's density basis (Gr_{barD})

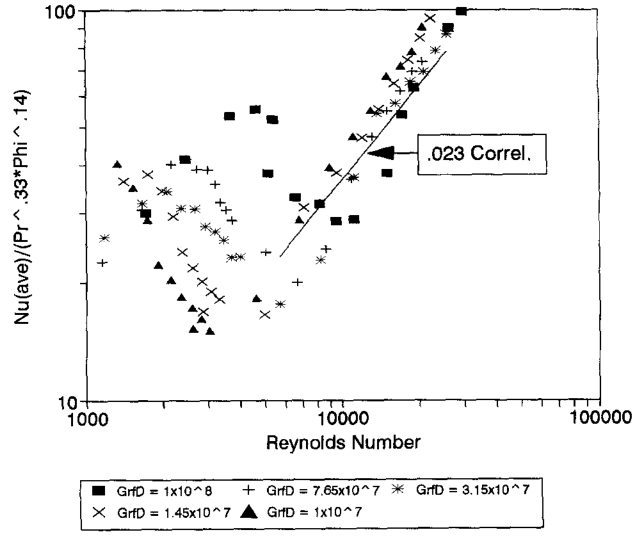

Figure 2 Aiding flow (upflow heating) heat transfer results — composite of runs with different Grashof numbers resulting from different wall heating (different steam pressures)

Asymptotic limits of the data

Mixed convection data need to be compared to some base case. The turbulent forced-flow-only correlation is a typical choice. From this, increases in heat transfer attributable to natural convection effects can be followed clearly. In this work, we prefer the Sieder–Tate equation with viscosity correction as the turbulent flow base case

$$\mathrm{Nu}/\mathrm{Pr}^{1/3}\phi_v^{0.14} = .023\ \mathrm{Re}^{0.8} \tag{1}$$

which is often referred to in shorthand as the ".023 equation." For heating situations, the viscosity correction factor is often folded into a slightly different power for Pr than the above. The correlation of Swanson and Catton (1987) for opposing flow shows this preference; the Jackson et al (1989) correlation uses the Petuckov–Kirillov relationship, which gives numerical results only slightly different from the Sieder–Tate. In any event, these base cases are all reasonably comparable.

Martinelli et al (1942) and McAdams (1954) both show an asymptotic relationship for CWT conditions at low Reynolds numbers. This is a result of losing ΔT driving force at the exit, and consequently would not be expected to hold in experiments with UHF at the wall (this maintains ΔT driving force). The equation takes various forms, but fundamentally it is as follows:

$$\mathrm{Nu}_{\mathrm{asymptote}} = (2/\pi)\ \mathrm{Gz} \tag{2}$$

where Gz is the well-known Graetz number. When Re is used as an independent parameter, as we prefer, the asymptote is Pr number dependent,

$$\mathrm{Nu}_{\mathrm{asymptote}} = 0.5\ \mathrm{RePr}\ D/L \tag{3}$$

which translates to

$$\mathrm{Nu}_{\mathrm{asymptote}}/\mathrm{Pr}^{1/3}\phi_v^{0.14} = 0.5\ \mathrm{Re}\ \mathrm{Pr}^{2/3}\ D/L \tag{4}$$

in the equivalent Sieder–Tate style formulation.

Heat transfer results for upflow heating

The upflow heating results (Nusselt number as a function of Reynolds number) are shown as a composite in Figure 2. As Grashof number is increased, the enhancement in heat transfer caused by mixed convection increases, as expected. We were not able to get uniform heating of the tube at steam pressures less than about 30 kPa absolute, and the lowest approach temperature we were able to achieve was about 40°C. At this level, the Grashof number is still rather high, and natural convection effects are still quite prominent, giving Nusselt numbers about half an order of magnitude higher than the laminar forced-convection correlation.

We can see from Figure 2 how the data approach both limiting regions; the turbulent region at high Reynolds numbers and the asymptote region at low Reynolds numbers. In the low Reynolds number region, high temperatures exist in all cases, and the Pr differences between the four runs is small. For Reynolds numbers above 12,000, the ".023 equation" predicts a bit low for the data, but this is typical, and others have used slightly different coefficients; e.g., .027, to get a better fit.

Several peculiarities are shown in the upflow heating case, which have also been noted by other investigators. (Poskas et al. 1994; Petuckov and Nolde 1959) There is a "dip" below the forced flow line in all cases. Why the data should show such a "negative enhancement" as the turbulent mechanism takes over is not clear, and a satisfactory explanation is still lacking. Poskas et al. recommend treating this as a developing flow situation, (Zeldin and Schmidt 1972) but more work needs to be done. In the same paper Poskas et al. review all the different formulations of buoyancy parameter that have been used by various investigators to correlate the results of these kinds of investigations. The authors recommend a 2.5 power of Reynolds number for regions of increased buoyancy effect and a 4 power of Reynolds number for regions of moderate influence of buoyancy (impaired heat transfer). Figures 3 and 4 show the present data in both formats, but it seems the 2.5 power of Re correlates best for both cases with the present data. In comparison to Poskas et al., the data in Figures 3 and 4 show a minimum in Nu/Nu_T of about the same magnitude at about the same location. The present experiments were done with water in a tube under CWT conditions; those of Poskas et al. were done with air in a flat channel with UHF conditions. Therefore, their data do not show a third region (which corresponds to the asymptote region) in Figures 3 and 4 or a maximum in enhancement factor, Nu/Nu_T, where Nu_T is computed from Equation 1.

The buoyancy parameter depends on Gr_q, which is a modified

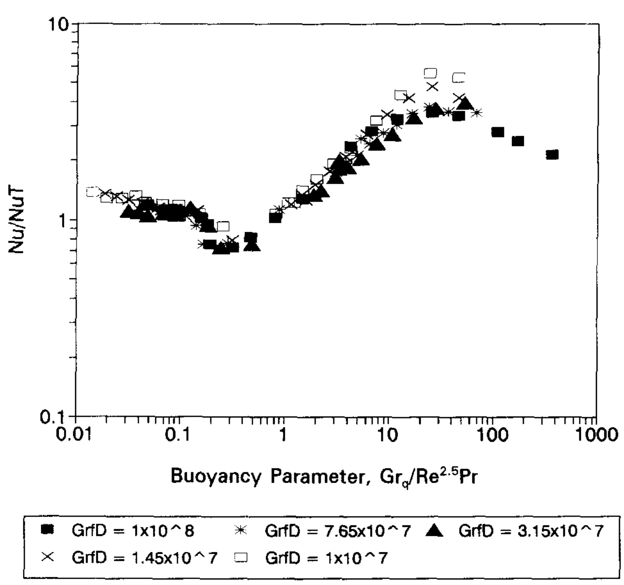

Figure 3 Enhancement factor as a function of buoyancy parameter, $Gr_q/Re^{2.5}Pr$

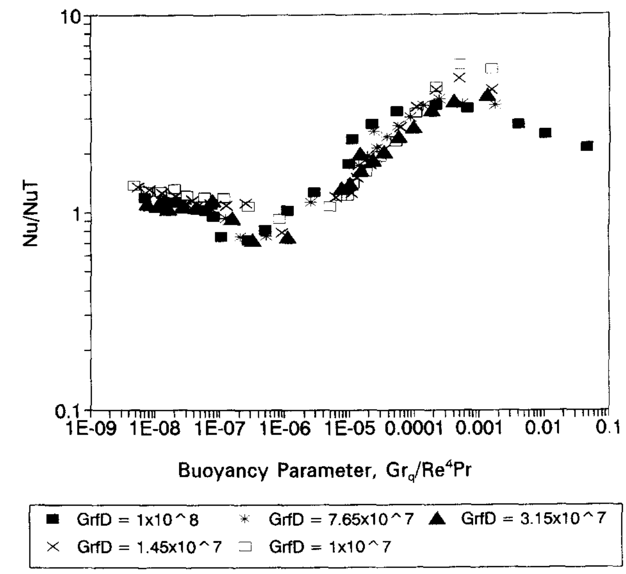

Figure 4 Enhancement factor as a function of buoyancy parameter, Gr_q/Re^4Pr

Grashof number equivalent to Gr × Nu. This eliminates the need to measure wall temperature in constant heat flux experiments and puts the heat flux directly into the Grashof number. This is not necessary for CWT conditions, but to compare data at the two boundary conditions, the same basis must be used. There is still considerable debate as to the best form of this buoyancy parameter, but the 2.5 power in Re seems best for all regions here, which cover a very wide range.

We can divide the data into various regions for purposes of correlation, as others have done (see Poskas et al. 1994). From Figure 3, at low-buoyancy parameter (below about 0.01), the enhancement factor should be 1.0. The data in Figure 3 are somewhat higher, because the Sieder-Tate equation was used as the base case; this predicts a bit low, and, thus, the enhancement factor is a bit high. For the region of impaired heat transfer from buoyancy parameter 0.01 to about 0.35, the enhancement factor is correlated by fitting the data; $Nu/Nu_T = 0.69(Gr_{bD}Nu/Re^{2.5}Pr)^{-0.16}$. For the region of increasing buoyancy effect from buoyancy parameter 0.35 to about 35, we could use $Nu/Nu_T = 1.15(Gr_{bD}Nu/Re^{2.5}Pr)^{0.4}$. For buoyancy parameter greater than about 35, the asymptote region comes into play, and the data are correlated by $Nu/Nu_T = 7.62(Gr_{bD}Nu/Re^{2.5}Pr)^{-0.28}$.

The data in Figure 3 do not match exactly with the data of Figure 10 in Poskas et al. (1994), but Grashof number was calculated on a different basis in each case. In Figure 3, Grashof number is calculated on the basis of fluid properties at the bulk average temperature (this would carry an additional subscript "bD", but for reasons of clarity it has been omitted). In Poskas et al., the Grashof number is calculated on Jackson's integrated density basis. Table 1 gives the translation for the present data. Applying an average factor of 1.8 brings the present data into very good agreement with the data of Poskas et al. for the region of increasing buoyancy effect (middle region of Figure 3).

Upflow/downflow comparison

Figures 5–8 show the comparison between an individual upflow heating run and its equivalent downflow heating conditions (same Grashof number). There is a quite pronounced negative slope in the upflow heating data that does not appear in the downflow case. The aiding flow results seem to be shifted both vertically and horizontally as Gr is reduced, but the opposing flow data seem to be shifted vertically only.

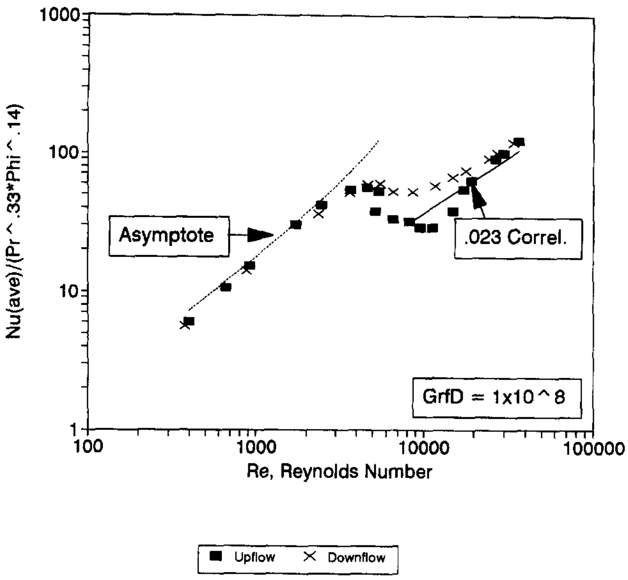

Figure 5 Upflow and downflow heating data compared at same Grashof number; shell-side steam pressure = 205 kPa ($Gr_{fD} = 1 \times 10^8$)

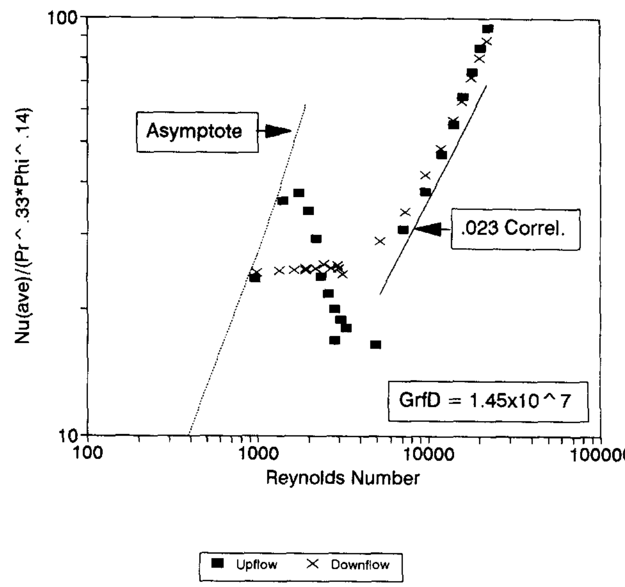

Figure 7 Upflow and downflow heating data compared at same Grashof number; shell-side steam pressure = 33.6 kPa ($Gr_{fD} = 1.45 \times 10^7$)

Figures 5–8 also show a crossover phenomenon between the upflow data and the downflow data. There is a different crossover with respect to the forced flow correlation, as discussed above. At the higher Grashof numbers, the downflow heating data are above the upflow heating data, but as Gr is reduced, the downflow heating data seem to be much more strongly influenced by Grashof number change than the upflow heating data, and this produces the crossover. The downflow data soon become partly above and partly below the upflow data. Presumably, turbulence at the shear layer is controlling for downflow, and this is reduced rather uniformly as Gr is reduced. However, such is not the case for upflow, which lacks turbulence and a shear layer. Herbert and Sterns (1972) also show a direct comparison of upflow and downflow data at the same Grashof number, but do not show the crossover or the asymptote, because the Reynolds number was not taken low enough. We can see, however, the same trend.

It also seems that the asymptote must play a very significant, and not previously appreciated, role in the upflow situation. The crossover could possibly be the result of a stabilization of the flow, which is known to occur for aiding flow conditions, (Scheele and Hanratty 1963; Scheele et al. 1960; Hanratty et al. 1958) so that the data follow the asymptote further into the Reynolds number range (higher flow rates) than the equivalent downflow situation.

The practical significance of this is clear, but unexpected; i.e., from a heat transfer standpoint, upflow heating can be more efficient than downflow heating. This results from the peculiarities of mixed convection flows. As parametric studies are broadened, perhaps we have not seen the last of these surprises. When the flow is in a certain Reynolds number range (below about

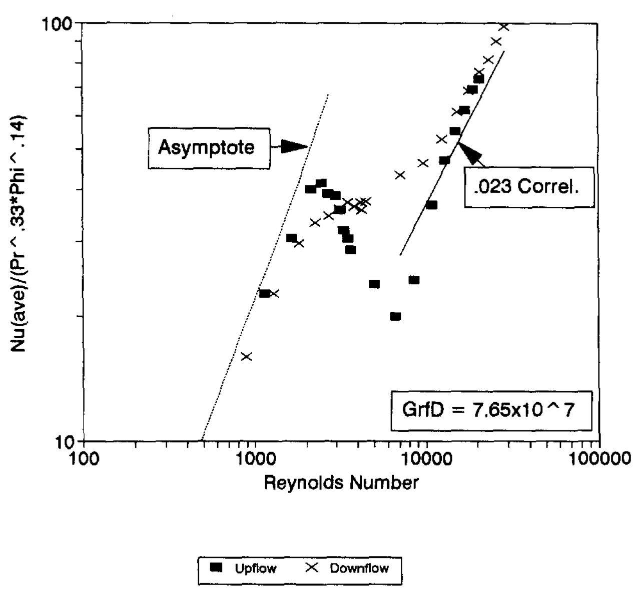

Figure 6 Upflow and downflow heating data compared at same Grashof number. Shell-side steam pressure = 99.6 kPa ($Gr_{fD} = 7.65 \times 10^7$)

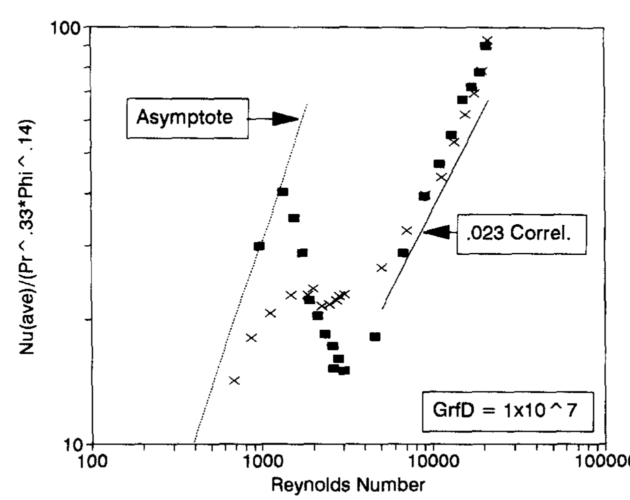

Figure 8 Upflow and downflow heating data compared at same Grashof number; shell-side steam pressure = 30.2 kPa ($Gr_{fD} = 1.00 \times 10^7$)

2500 from present data), and heating conditions give a Gr_{fD} below 1×10^8, this phenomenon occurs.

Conclusions

Within the limits of our experimental equipment, we have shown quantitatively how heat transfer in the mixed convection region for flow in a vertical tube is altered by changing Grashof number. Opposing flow generally gives higher heat transfer enhancement than aiding flow in the mixed convection zone. However, as Gr is reduced, a crossover phenomenon occurs, where the above is true for the upper part of the Reynolds number range for mixed convection only. There is obvious practical significance to these results, but a clear explanation of why this should happen awaits further study.

The data of the present investigation taken under CWT conditions, when plotted as enhancement factor versus buoyancy parameter, seem to show the same trends as literature data taken under UHF conditions, with the exception of the asymptote region.

Acknowledgments

The author would like to thank the following students for their assistance with data collection and reduction: Matt Wojnovich, Brian M. Shaughnessy, Steven F. Farnan, Dennis J. Howe, and Timothy F. Kelly.

References

Hanratty, T. J. Rosen, E. M. and Kabel, R. L. 1958. Effect of heat transfer on flow field at low Reynolds number in vertical tubes. *Ind. Eng. Chem.*, **50**, 815–820

Herbert, L. S. and Sterns, U. J. 1972. Heat transfer in vertical tubes — Interaction of forced and free convection. *Chem. Eng. J.*, **4**, 46–52

Holman, J. P. 1990. *Heat Transfer*, 7th ed., McGraw-Hill, New York

Ianello, V., Suh, K. Y. and Todreas, N. E. 1988. Mixed convection friction factors and Nusselt numbers in vertical annular and subchannel geometries. *Int. J. Heat Mass Transfer*, **31**, 2175–2189

Jackson, J. D., Cotton, M. A. and Axcell, B. P. 1989. Studies of mixed convection in vertical tubes: A review. *Int. J. Heat Fluid Flow*, **10**, 2–15

Joye, D. D. 1995. Comparison of correlations and experiment in opposing flow, mixed convection heat transfer in a vertical tube with Grashof number variation. *Int. J. Heat Mass Transfer*

Joye, D. D., Bushinsky, J. P. and Saylor, P. E. 1989. Mixed convection heat transfer at high Grashof number in a vertical tube. *Ind. Eng. Chem. Research*, **28**, 1899–1903

Marcucci, B. J. and Joye, D. D. 1985. Experimental study of transitions in mixed-convection vertical upflow heating of water in tubes. In *Fundamentals of Forced and Mixed Convection*, F. A. Kulacki and R. D. Boyd (eds.), ASME New York, 131–139

Martinelli, R. C., Southwell, C. J., Alves, G., Craig, H. L., Weinberg, E. B., Lansing, N. F. and Boelter, L. M. K. 1942. Heat transfer and pressure drop for a fluid flowing in the viscous region through a vertical pipe. *Trans. Am. Inst. Chem. Eng.*, **38**, 493–530

McAdams, W. H. 1954. Heat Transmission, 3rd ed., McGraw-Hill, New York, 229–235

Petuckov, B. S. And Nolde, L. D. 1959. Heat transfer in the visco-gravitational flow of liquid in pipes. *Teploenergetika*, **6**, 72–80 (in Russian)

Poskas, P., Adomaitis, J. E., Vilemas, J. and Bartkus, G. 1994. Development of turbulent heat transfer over the length of vertical flat channel under a strong influence of buoyancy. *Proc. 10th Int. Heat Transfer Conference*, Paper 12-NM-26 G. F. Hewitt (ed.), IChemE, Rugby, UK, 555–560

Scheele, G. F. and Hanratty, T. J. 1963. Effect of natural convection instabilities on rates of heat transfer at low Reynolds numbers. *AIChEJ*, **9**, 183–185

Scheele, G. F., Rosen, E. M. and Hanratty, T. J. 1960. Effect of natural convection on transition to turbulence in vertical pipes. *Can. J. Chem. Eng.*, **38**, 67–73

Swanson, L. W. and Catton, I. 1987. Surface renewal theory for turbulent mixed convection in vertical ducts. *Int. J. Heat Mass Transfer*, **30**, 2271–2279

Symolou, P. D., Todreas, N. E. and Rohsenow, W. M. 1987. Criteria for the onset of flow recirculation and onset of mixed convection in vertical rod bundles. *J. Heat Transfer*, **109**, 138–145

Zeldin, B. and Schmidt, F. W. 1972. Developing flow with combined forced-free convection in an isothermal vertical tube. *J. Heat Transfer*, **94**, 211–223